

Rectilinear Wide-angle Lenses and Fish-Eye (All Sky) Lenses

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Photographic lenses are usually designed so that straight lines in a scene are represented by straight lines in the image. Thus a rectangle in a plane perpendicular the axis of the lens will be imaged as a rectangle. Lenses that achieve this effect are said to be rectilinear. Wide angle lenses with focal lengths much less than the diagonal length of the image are difficult to construct. With such lenses barrel distortion, where vertical and horizontal lines tend to bow out away from the center of the image, is common; and corrections become more difficult as the focal length decreases. For example, full frame 35mm cameras have an image diagonal of 43.3mm, and the smallest practical focal length for highly corrected rectilinear lenses turns out to be about 15mm.

The problem of obtaining wide fields of view is especially severe for owners of the current crop of consumer oriented single lens reflex cameras (SLR's) which have detectors that are smaller than film frames in full frame 35mm cameras. For example, full frame images are 24mm by 36mm and cameras with a crop factor of 1.6 have detectors that are only 15.1mm by 22.7mm. The result is that the effective focal lengths of lenses on these digital SLR's are 1.6 times their true focal lengths. This is a boon for telescopic photography where effective focal lengths are boosted so that a 200mm lens acts like a 320mm lens. However, a 28mm wide angle lens suddenly appears to be 45mm – not very wide at all, and even a super wide 20mm lens acts like a wimpy 32mm semi-wide angle lens.

The appearance of “crop factors” has led photographers to seek much shorter focal lengths in an attempt to recapture wide fields of view. But that is the problem. With a crop factor of 1.6 we need a 12.5mm lens to recover the angle of view that the full frame camera gives with a 20 mm lens. As noted above, a 12mm focal length is beyond the range of practical rectilinear lenses that can produce full frame images. One way around this problem is to design special “digital lenses” that are only corrected for the smaller cropped image area. This, of course, results in a set of incompatible lenses that will be useless for current and future full frame cameras. Another problem with producing smaller wide angle lenses for SLR's with small detectors is that the distance between the rear element of the lens and image plane cannot be reduced very much because of the moveable mirror that still must be accommodated. Alternative routes for smaller wide angle lenses are either to scale down sizes of the lens mount and mirror to reduce the lens to detector distance or to resort to an electronic view finder and do away completely with the moveable mirror. The latter possibility may be the preferred one for electronic view finders that provide sufficient resolution.

Fortunately, fish-eye lenses provide another alternative for SLR users who seek ultra wide angle views.^{1,2} Fisheye lenses are, of course, not new; but computer conversion of fisheye to rectilinear images certainly is.³⁻⁵ Furthermore, the 1.6 crop factor often yields fisheye images that need little correction. The mapping of a scene into a flat image is fundamentally different for rectilinear and fisheye lenses. The horizontal

field of view (HFOV) of a rectilinear lens in degrees is easily determined from a geometric construction in which straight lines representing light rays connect the center of the lens with the extreme ends of the image as shown in Fig 1.

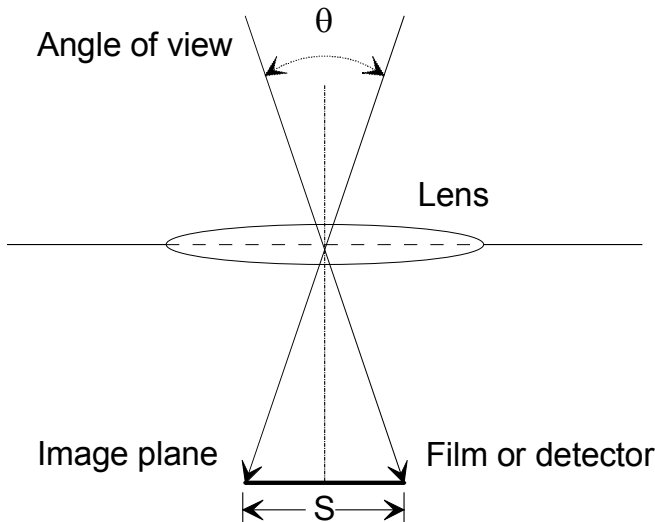


Figure 1

The assumption is that rays from a distant object pass through the center of the lens and proceed to the image without deviation just as they would in a pinhole camera. The result of this exercise is the equation $HFOV = (360^\circ / \pi) \arctan [S / (2 \cdot FL)]$ where S is the size of the film plate or detector and FL is the focal length. For full frame 35mm cameras the horizontal image width is 36mm, and we obtain the values shown in Table 1. The Canon EF lens specifications table reports fields of view for the diagonal instead of the horizontal orientation. The angles listed in the Canon table can be approximately reproduced by using 43.3mm as the film width (S) in the HFOV equation. Similarly, the HFOV for cameras with 1.6 crop factors can be obtained by replacing the detector width with 22.7mm.

Table 1

Focal Length (mm)	HFOV (degrees)	Focal Length (mm)	HFOV (degrees)
15	100.4	50	39.6
20	84	100	20.4
28	65.5	200	10.3
35	54.4	300	6.9

In striking contrast to rectilinear lenses and the HFOV predictions, fish-eye lenses yield circular images in which the distance from the center is simply related to the angle of view. For the ideal (angular) fish-eye lens, the distance is directly proportional to the angle of view. Therefore, if a ray makes an angle of θ with the axis of the lens, the corresponding point in the image is a distance R from the center where $R = a\theta$ and “a” is a constant. Standard fish eye lenses for 35mm cameras give 180° fields of view and are

available in two types. Hemispherical fish-eyes lenses with focal lengths in the range 6 to 8mm fit the complete 180° field of view disk into the narrow dimension of the image, while full frame fish-eyes with focal lengths of 14 to 16mm spread the hemispherical image across the film or detector diagonal. For full frame fish-eye lenses with R in mm and θ in degrees, the proportionality constant is $a = 0.24\text{mm}/\text{deg}$. Therefore, with a 1.6 crop factor the HFOV would be about 90°. A rectilinear lens would need a focal length of about 11mm to match this. Even with a full frame 35mm camera the 90° HFOV would require a 18mm ultra wide angle lens. Actually, there is no problem with going even wider with a fish-eye lens. A Nikkor fish-eye lens has been produced with a focal length of 6mm and a 220° field of view.

As noted, computer correction of barrel distortion in fish-eye lenses is now available. Consider the image in Fig. 2 obtained with a full frame (FL=15mm) fish-eye lens (Sigma EX) on 35mm film.



Figure 2

The yellow rectangle defines the area the image that would be obtained with a crop factor of 1.6. Notice that each small segment of the image has small distortion even though the overall image shows strong barrel distortion. The effect is similar to that of looking through a transparent hemisphere divided into many sections, freezing the scene observed into each segment, and then flattening the hemisphere onto a plane. The apparent distortion results from the large angles displayed. By design the angular spread from corner to corner diagonally is 180° and the horizontal and vertical fields of view are approximately 150° and 92°, respectively. Two horizontal lines will naturally appear to approach each other at the ends as required by perspective.

Now we convert the fish-eye image to a rectilinear image by adjusting the distance of each image point from the center. This conversion was performed with Panorama Tools written by Helmut Dersch.³ The result is shown in Fig. 3 for full frame and 1.6 crop factor cameras.



Figure 3

It is, in fact, the rectilinear image that conflicts with reality at very wide angles by attempting to counteract perspective. Recall that parallel lines are supposed to converge at infinite distance. The distortions produced by ultra wide rectilinear lenses are perhaps more disturbing than the fish-eye distortions. However, in both cases we are attempting to represent a wide view of the 3D world with a planar image. It is analogous to the problems that map makers face in representing the globe with flat maps.

Historical note: The term fish-eye view was introduced into photography by Robert Wood, who was an optical physicist at Johns Hopkins University. In his text *Physical Optics* he invited the reader to consider the world outside the water as viewed by a fish.⁶ Directly overhead, objects would be seen clearly; but away from the vertical direction there would be distortion. Also, beyond a critical angle the surface of the water would be a mirror because of total internal reflection. The effect can be quantitatively understood by considering refraction of rays of light at the air/water interface as illustrated in Figure 4.

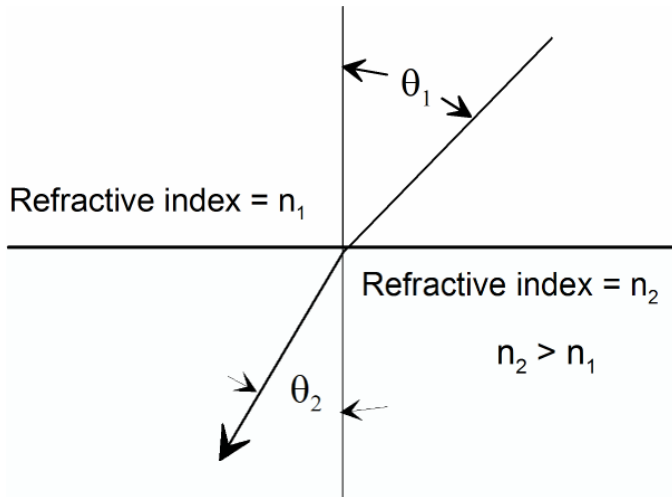


Figure 4

According to Snell's law, $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$, and since $n_1 \approx 1.0$ for air and $n_2 \approx 1.33$ for water, the angle of incidence θ_1 is always greater than the angle of refraction θ_2 . Also, we find that when $\theta_1 = 90^\circ$, the angle θ_2 of the refracted ray reaches its maximum value of about 49° . Figure 5 shows how the refracted rays in the water, that are restricted to angles less than 49° , map out the full 180° view above water.

Fish-eye view

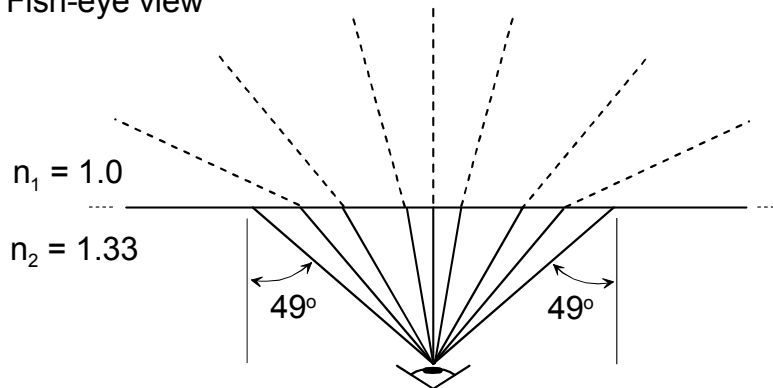


Figure 5

The human eye is not constructed to be able to see this effect very well, but Prof. Wood pointed out that we can easily photograph it. He went on to construct a pinhole camera^{7,8} filled with water and demonstrated that it could produce photographs with the full 180° hemispheric view. Four of these photographs are reproduced in his text.

In Wood's pinhole water tank camera, the distances from the center of the image are simply related to the angle of view with 0° at the center and 180° at the edge as illustrated in Figure 6.

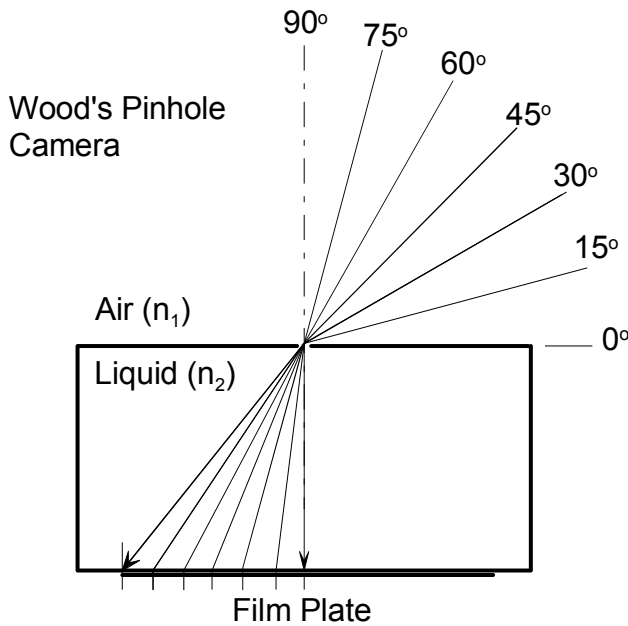


Figure 6

The absolute distances, and hence the scale, depend on the distance from the pinhole to the film plate and the refractive index of the fluid in the tank.

Reference List

- 1) Kumler, J and Bauer, M. Fisheye lens designs and their relative performance. 2000. Lens and Optical System Design and Engineering Conference of the SPIE Annual Meeting. <http://www.coastalopt.com/news/ne000804.asp>
- 2) Kingslake, R. A history of the photographic lens. 1-334. 1989. Academic Press.
- 3) Dersch, H. Panorama Tools. 2001. <http://www.path.unimelb.edu.au/~dersch/>
- 4) Turkowski, K. Making environmental maps from fisheye photographs. 7-4-1999. <http://www.worldserver.com/turk/quicktimevr/fisheye.html>
- 5) Bourke, P. 2001. <http://astronomy.swin.edu.au/~pbourke/projection/fisheye/>
- 6) Wood, R. W. Physical Optics. 2nd Ed., 1-705. 1911. Macmillan.
- 7) Kingslake, R. Lenses in Photography. 1-246. 1951. Garden City Books.
- 8) Walker, J., The pleasure of the pinhole camera and its relative the pinspeck camera, *Sci.Am.* **1981**, 245, 192-200.